Kripkean Satisfaction For Unrestricted Higher-Order Languages

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The aim of my talk is to develop a Kripkean theory of truth (or rather satisfaction as it will become clearer later on) for unrestricted higher-order languages. A language is unrestricted if its quantifiers can range over absolutely everything there is. Whether this is possible is the matter of contention between generality absolutists and generality relativists (henceforth "absolutists" and "relativists"). Absolutists answer in the affirmative, relativists in the negative.

The set-theoretic domains of models in standard model theory are grist for the relativist's mill: Domains of models are sets, and since there is no set of all sets, there is no set of absolutely everything. As a result, model theory is inherently relativistic.

Recent works saw the development of absolutist-friendly analogues of model-theoretic notions: Rayo and Uzquiano (1999) and Rayo and Williamson (2003) have developed Tarskian accounts of truth and satisfaction for unrestricted first- and second-order languages, and have sketched how to generalize their accounts to unrestricted higher-order languages. The main idea is to formalize model-theoretical notions such as *interpretation* and *model* for a given object language as *predicates of a higher-order metalanguage*. The higher-order framework allows one to drop the assumption that domains are sets, thereby circumventing the set-theoretical size-restrictions. Without such restrictions, quantifiers can be interpreted as ranging over absolutely everything. Let us call this the RU-framework, where RU stands for Rayo and Uzquiano.

However, the Tarskian account of truth and satisfaction is based on a type distinction: Where $\mathcal{L}$ is an object language and $\mathcal{L}'$ a metalangauge, truth-in-$\mathcal{L}$ is formalized by an $\mathcal{L}'$-predicate Tr which applies only to $\mathcal{L}$-sentences. Since any sentence in which Tr occurs is by construction a $\mathcal{L}'$-sentence, truth predication cannot be iterated. A way to allow for iterations is by going to yet a further metalanguage $\mathcal{L}^*$ – a metametalanguage of $\mathcal{L}$ – which has a further truth predicate Tr^* that can be applied to $\mathcal{L}'$-sentences. But this just shifts the problem rather than solving it. We can now say "it is true that it is true that φ", but only when φ is an $\mathcal{L}$-sentence and not when it's an $\mathcal{L}'$-sentence. A generalization of this idea leads to a hierarchy of metalanguages, each of which has its own truth predicate. But this seems highly ill-motivated with respect to natural language.

Furthermore, as Kripke has famously argued, in a typed hierarchy of languages, blind ascriptions such as "Everything Tarski said about truth is true" pose a prima facie conundrum: Without knowing what the highest type of Tarski's utterances about truth was this sentence cannot be modelled correctly. The shortcomings are caused by typing, and a type free account of truth and satisfaction in the spirit of Kripke's seminal work (1975) eludes both of them.

A first step in the development of a Kripkean notion of satisfaction for unrestricted languages has recently been taken by Rossi (forthcoming) in a truth-theoretical setting. Although the technique developed by Rossi is type free, it is only developed for first-order object languages. Consequently, there is no account of Kripkean truth for unrestricted higher-order languages available in the literature, and it is the primary aim of the present work to fill this lacuna.

There are good reasons to incorporate higher-order object languages in our semantic frameworks: The categoricity theorems of second-order logic, most famously the categoricity of second-order Dedekind-Peano arithmetic, crucially depend on fully interpreted second-order logic. I will take the categoricity theorems as the starting point of my talk, and argue that categoricity is a phenomenon that can be found in natural language. By a methodology that I call *inference-to-the-strongest-language*, I conclude that the mere fact that there is categorical talk about $\mathbb{N}$ in natural language suffices to motivate the incorporation of full higher-order logic in our formal frameworks.

Having established that, I will develop an absolutist-friendly formal framework for higher-order object languages. I will first identify the main features of the notions *model*, *domain*, and *interpretation* for higher-order object languages in the standard model-theoretic setting, and reformulate them in the RU-framework, thereby making them compatible with absolutism. All notions will be defined rigorously for nth-order object languages as predicates of an $(n+1)$th-order metalanguage.

In the next step, I will identify the main features of Kripke's model theoretic construction of an interpretation of the truth predicate. This is usually carried out in stages: We fix a higher-order base structure which interprets arithmetic and construct a set E which acts as the extension of the truth predicate. At the first stage, E is empty. Then, all valid base sentences, as well as all negations of invalid base sentences, are added to E. In the next step, E will be closed under strong Kleene evaluations, as well as truth predications. So if two sentences φ and ψ are in E, so is their conjunction. If a sentence φ is in E, so is its double negation. And similarly for truth predications: If φ is in E, so is "φ is true"; and if $\neg\varphi$ is in E, so is "φ is not true".

I will then define Kripkean satisfaction (which includes truth as a special case) as a predicate of an $(n+1)$th-order metalanguage for an nth-order object language. The predicate essentially mimics the construction of E in the RU-framework. Taken together, the base structure and the formalized satisfaction predicate define a natural and fully classical higher-order "model" for an arithmetical base theory augmented with a Kripkean truth predicate. However, unlike standard models, the construction in the RU-framework has quantifiers ranging over absolutely everything.

If time permits, I will introduce a higher-order version of Feferman's seminal axiomatization of Kripke's model-theoretic truth theory KF (see Feferman, 1991), which I call KF_n, and I will show that KF_n has natural models. Even though such natural models are usually taken to be set-sized, KF_n is also validated by absolutist-friendly higher-order models in the sense of the RU-framework.

I will conclude that absolutist-friendly analogues of model-theoretic notions are available for higher-order languages, even if satisfaction and truth are understood in the sense of Kripke (1975). Moreover, I will conclude that type free truth for higher-order languages can be axiomatized by KF_n. Finally, the framework is of philosophical significance as it combines several interesting features such as generality absolutism, being type free, and categorical talk about important mathematical structures.

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