Coherentist Structuralism: Structures as Thin Objects

In this article I pursue a position I will call *coherentist structuralism*. It is a combination view that unites the ontology of non-eliminative structuralism and the metaontology of coherentist minimalism. The idea is that non-eliminative structuralism has a natural ally in coherentist minimalism, and that the positions have certain traits that suggest they benefit from being combined. Moreover, I argue that the central claim of structuralism – that abstract structures exist – is given added justification by situating it within a broader philosophical framework; namely, by adopting metaontological coherentism. Attendant to pursuing this combination view, is a general elucidation of coherentism and the central notion of coherence. I reject previous attempts to construe coherence as a formally defined mathematical notion, and instead look to coherence theories in analytic epistemology. I argue that there are conceptual overlaps, especially regarding the emphasis put on non-linearity, so that the notion of coherence becomes philosophically informed. By providing nuance to how the notion of coherence can be attributed in various degrees and in different ways, the upshot is that coherence can be construed as an existence criterion fitting for non-eliminative structuralism.

In the first part, I explain what is meant by a metaontology for mathematics. I sketch two approaches to it, the historical roots of which stem from the Quine-Carnap dispute on the legitimacy of ontology and ontological commitment to abstract entities. Most metaontologies for mathematics are developed by ontologically realist positions, as the burden is on the realist to prove the existence of mathematical objects. It is the ontological realist that needs added justification for her views, and one way to go about that is to impose certain limiting requirements. If her ontological claims must submit to restrictive conditions, the claims will i) follow the same guiding principle, ii) become uniform, and iii) ensure that the ontology as a whole has a desired unity. Ontological claims thus receive qualification, making implicit theoretical underpinnings explicit, by articulating criteria for when we allow mathematical entities to exist.

The second part is on metaontological minimalism and *thin objects*, and how these ideas are instances of a deflationary approach to metaontology. Metaontological minimalism is a metaontological view that recently has seen some traction within a realist setting, notably defended by Øystein Linnebo (2018). Metaontological minimalism does not support a minimal ontology, on the contrary, generous ontological views are very much compatible with this metaontological stance. It is rather the criteria for one's ontological commitments that are minimal. This allows for what Linnebo calls *thin objects*, where the idea is that an object is considered *thin* if it does not make substantial demands on the world. While a pure mathematical object is thin in an *absolute* sense (viz., sets, numbers, etc.), there are objects that are thin only in a *relative* sense, as well. An example is the set of two trees, where the set does not make any *further* substantial demands on the world, other than that of the spatio-temporal make-up of the trees in question.

In the third part, I outline what coherentist minimalism entails as a metaontological position, and I provide different explications of the notion '*coherence*'. Coherentism is the view that given the *coherence* of a mathematical theory, the existence of the objects described by the theory in question is ensured. Coherentist approaches have roots in Hilbert's views, but have more recently been defended by Shapiro (1997). Coherentism is *minimalist* insofar that what is needed for an object to exist is very little. The existence of the intended object depends on the coherence of the theory that describes it, i.e., if the theory is coherent, the objects described by the theory exist. As such, coherentism constitutes another approach to *thin* objects. The existence of objects resulting from this coherence is thin because their existence does not put any *further* metaphysical demands on the world, other than that of the theory providing their description (and thereby their existence).

Crucial to Shapiro's structuralism, is the existence axiom for structures and their positions, viz., the *Coherence Principle*: "**Coherence**: If **Φ** is a coherent formula in a second-order language, then there is a structure that satisfies **Φ**" (1997:95). Shapiro uses the coherence of a formula to assert that there exists a structure that satisfies the formula in question. This is clearly a position that commits itself to the existence of abstract structures *by way of coherence*. And while Shapiro uses the Coherence Principle as an existence criterion, he stops short of developing a larger metaontological framework. Moreover, what the notion 'coherence' really means, remains unclear. As Shapiro concedes: "The problem, of course, is that it is far from clear what 'coherent' comes to here", and also: "Coherence is not a rigorously defined mathematical notion, and there is no noncircular way to characterize it" (Shapiro 1997:95, 13).

The notion of coherence thus remains woolly and in need of clarification and philosophical merit. This opens up the possibility of looking somewhere else than to philosophy of mathematics. Coherence finds its most developed form in theories of justification. Coherence theories of justification in analytic epistemology is often considered as the refusal of one of the main theses of foundationalism. Foundationalism is linear, one-directional and reductionist in character, where some beliefs have epistemic priority over others. This is not the case for coherentism. In coherentism beliefs lend mutual support and justification to each other, as long as they all belong to the same system of beliefs. John Bender characterises epistemological coherentism as the view where "no empirical beliefs enjoy epistemic priority, and all rely for their justification on their connection to, or membership in, the body of other things believed or accepted" (Bender 1989:1). Epistemological coherentism thus advocates a picture of justification that emphasises its opposition to linear accounts, e.g., foundationalism. Moreover, it provides a distinction between *systemic* and *relational* coherence, in order to characterise how the system's internal arrangement, and how the system as a whole, provides justification in different ways (see Bender (1989)). This gives us a more nuanced picture of non-linear transmissions of justification in terms of coherence.

In the fourth part, I move on to the combination view of coherentist structuralism. I apply the insight begot from my discussion on coherence theories of justification and the conceptual distinction between systemic and relational coherence to our case of non-eliminative structuralism. A central tenet of structuralism is the idea that mathematical objects are *incomplete*, as their natures are exhausted by their mathematical context, i.e., the structure to which they belong. This leads to objects standing in an ontological dependence relationship, formulated by Linnebo (2008) as the Dependence Claim, which consists of two tenets. The first is that a mathematical object depends on the structure to which it belongs. The second tenet is that a mathematical object also depends on the other objects belonging to the same structure. This means that the natural number 2, for example, not only ontologically depends on the natural number structure, but also on the numbers 3, 6, 101, etc.

I argue that this twofold dependence relationship finds a correspondence in the distinction between systemic and relational coherence. If we can reformulate this dependence relationship in terms of systemic and relational coherence, we are one step further to articulate an acceptable existence criterion for metaontological coherentism. I further argue that if the distinction between systemic and relational coherence can inform our metaontological coherentist framework, developing coherentist structuralism holds real promise, as we might advance our understanding of how we think of structures and their objects, and what it takes for them to exist. On the resultant combination view, with coherence as the threshold for existence, structures are conceived of as thin, thus making them metaphysically lightweight. This allows for a thinner realism, as opposed to a robuster realism *à la* platonism in mathematics.

References

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