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The evolution of the proofs-as-programs paradigm through realizability

Realizability was invented by Kleene in 1945 to formalize the view that proofs in mathematics should correspond to computable functions. Thus Realizability aims at extracting the computational content of mathematical proofs by establishing a correspondence between proofs and programs, the Curry-Howard Isomorphism is a famous exemple of such a correspondence. For many years, it seemed that the proofs-as-programs paradigm should be limited to intuitionistic logic, then the work of Griffin (1992) on the typeability of the lambda_C calculus made possible to extend the proofs-as-programs correspondance to classical logic. J-L. Krivine pushed this program even further by inventing a technique for building realizability models for Zermelo-Fraenkel set theory, and he recently proved that even the Axiom of Choice and forcing-consistent principles can be realized. In a joint work (in progress) with Geoffroy and Krivine, we show that even certain large cardinals axioms can be realized. Developed within research in constructive mathematics, the proofs-as-programs view has evolved to include classical logic first, then set theory including the axiom of choice (which has been historically considered as the paradigm of non-constructive mathematics), then even large cardinals; we will discuss the implications of these results in the philosophical conception of proofs in mathematics.

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